

# Report on solar radiation modelling

## Solar energy on the four external walls and the tambour of the Church of Nativity of Virgin Mary, Gelati

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### PART I. THE SITE AND THE MODEL SIMULATION

#### The model simulation

A model simulation has been made to calculate the input of solar energy supplied during the daytime, month by month to the external surface of the four facades and the tambour of the Church of Nativity of Virgin Mary, Gelati.

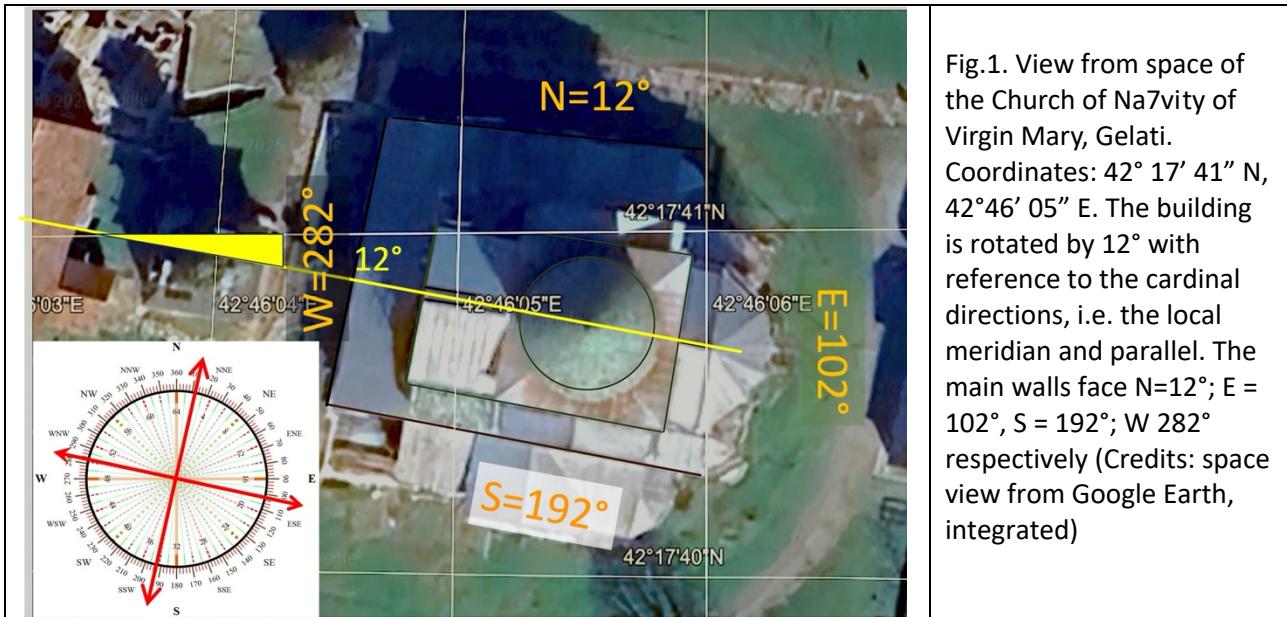
The calculations are based on True Solar Astronomical Time, which fixes noon (12 o'clock) at the instant of the solar culmination, when the Sun reaches its maximum height over the local horizon and passes through the local meridian. This time is the same as in sundials and is used in astronomy to make calculations easier. Over the calendar year, this time reaches small departures with respect to the official time of the clock, but this has no influence on the calculation of the incident radiation.

The model calculation considers the apparent motion of the Sun on the ecliptic (the yearly trajectory of the Sun in the celestial sphere), and then the daily apparent motion of the Sun above the local horizon determined by its height and hourly angle with respect to the local meridian. Given the trajectory of the solar rays, the model calculates the effective energy, after a fraction has been absorbed when crossing the atmosphere.

After, the model calculations consider how the incoming energy is distributed on the horizontal plane and on vertical surfaces at the four selected orientations, as well as on the tambour.

#### The orientation of the Galati Monastery

The actual orientation of the Church has been verified on the images from space of Google Earth which include the local meridians and parallels, as well as their coordinates. For clarity and ease of understanding, the building facades have continued to be named after the four cardinal points, while the incoming solar radiation has been calculated for the exact orientation of each facade. Although this rotation is apparently small (i.e. 12° roof rotation, as shown in Fig.1) in certain seasons it causes considerable differences. For instance, two correctly oriented East and West sides should receive the same amount of energy, but at different times of the day: morning and afternoon. With 12° rotation, the difference might appear negligible in summer but causes substantial differences in winter.



The misalignment of the Church is not casual, because the Church of St George, the Church of the Nativity of the Virgin Mary, and the Church of St Nicholas, Gelati, are aligned between them, forming an angle of some  $10^{\circ}$  with respect to the local parallel (Fig.2). The Church of Nativity, and especially St George, have a further, small rotation. Sunrise and sunset fall in this alignment twice a year: on March 3<sup>rd</sup> and October 9<sup>th</sup>.

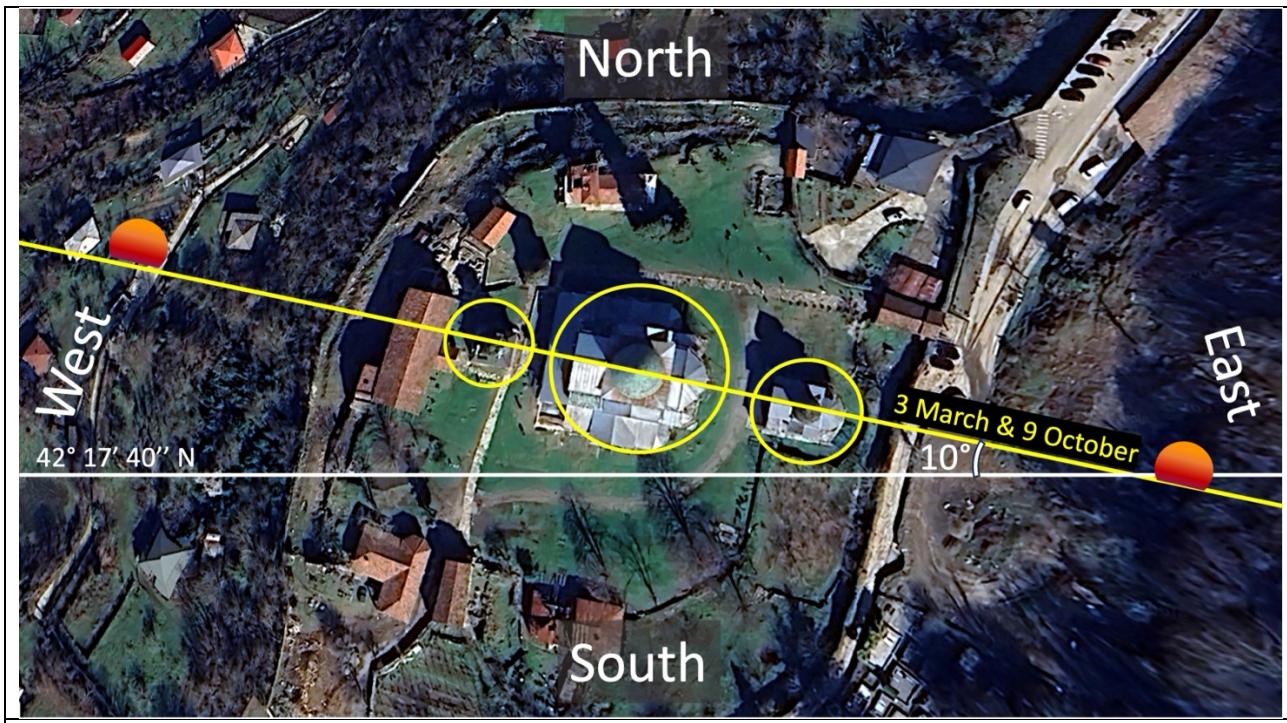


Fig.2. The three Churches of the Monastery are aligned between them (yellow line), forming an angle of some  $10^{\circ}$  with the local parallel (white line). Sunrise and sunset fall in this alignment on March 3<sup>rd</sup> and October 9<sup>th</sup>. (Credits: space view from Google Earth, integrated)

In addition to the above departure from the traditional East-West orientation, the Gelati Monastery presents another problem. On the eastern and southern side, the Monastery is surrounded by a sloping and uneven mountain relief which shields the early solar radiation for some time after sunrise (Fig.3).

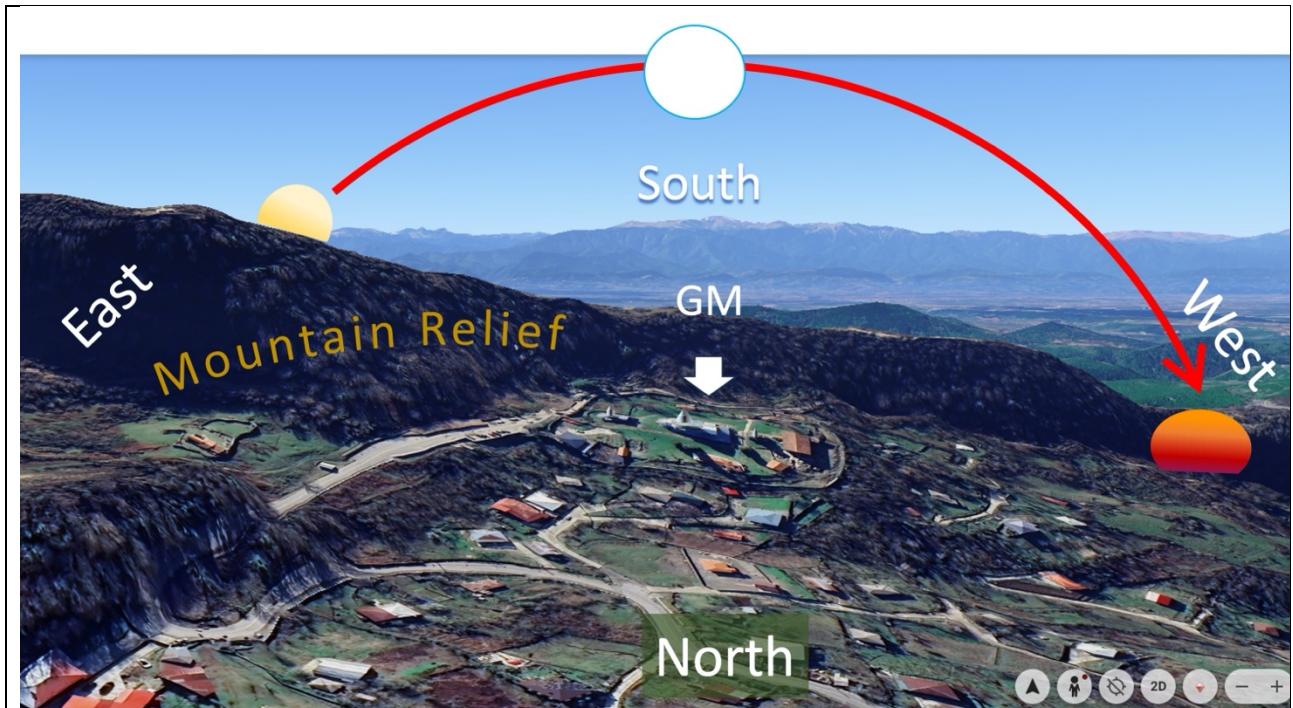


Fig.3. The Gelati Monastery (GM) and its surrounding landscape, seen from North. A sloping and uneven mountain relief lies on the eastern and southern sides. At sunrise, it shields the early solar radiation for some time until the Sun emerges from behind the relief. (Credits: space view from Google Earth, integrated)

### The shadow cast by the mountain relief

The solar radiation has been calculated with the astronomical formulae in which the horizon is considered free. After, with a local topography with the indication of the contour line levels (Fig.4), it has been possible to calculate where, and when, the mountain slope shadows the Church of Nativity and the other buildings of the Monastery.

The focal point has been considered in the floor in the centre of the Church of the Nativity, at the local ground elevation, i.e. 400 m above mean sea level. The horizontal distance between the Church and the 450 m and 500 m contour lines has then been measured on Google Maps.

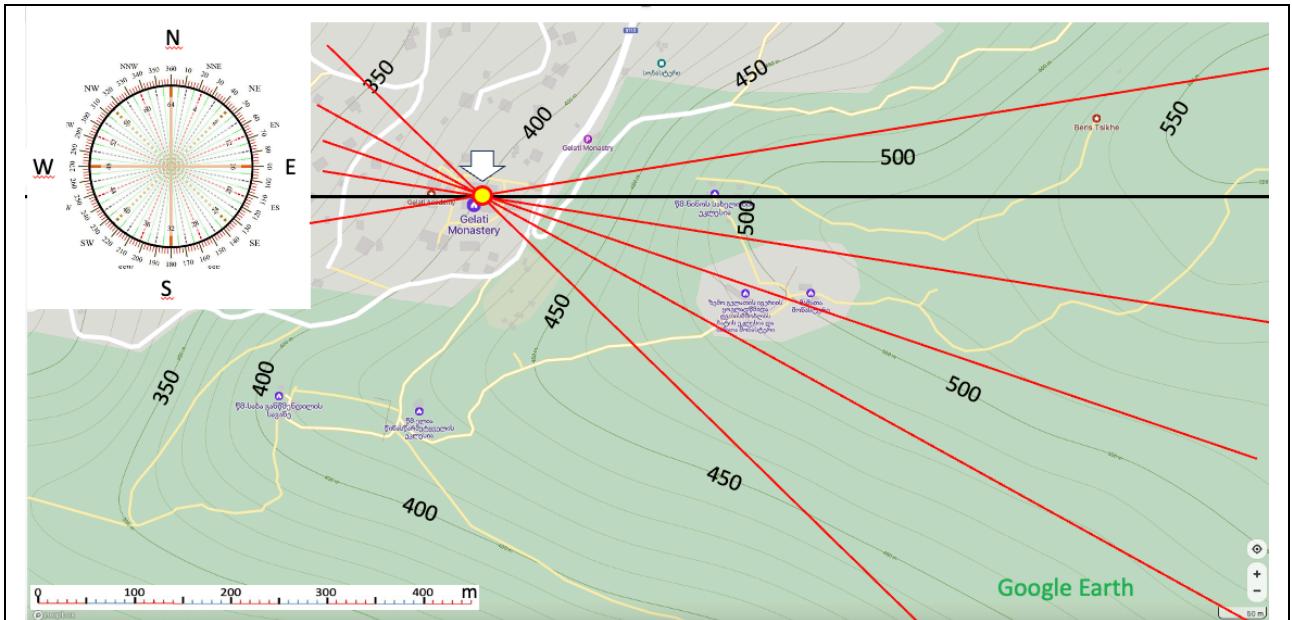


Fig.4. Planimetry of the mountain region around the Gelati Monastery (yellow dot with arrow). Contour line elevations are expressed in m. The horizontal black line is the local parallel, and the red line is the orientation of the three Churches. (Credits: Google Earth, integrated)

From these data, the blind spot caused by the mountain relief responsible for the early morning shadow has been calculated as a function of the solar azimuth angle  $A_{\odot}$  observed from the centre of the Church. The Sun remains obscured behind the mountain relief, until its elevation  $H_{\odot}$  over the local horizon exceeds the angle of the mountain blind spot (BA in Fig.5). When the solar elevation exceeds the blind threshold, the church is hit by solar beams, starting from the dome and the roof. Of course, the critical elevation  $H_{\odot}$  as well as the blind spot BA change day by day, during the calendar year. The figure shows an example of two different days.

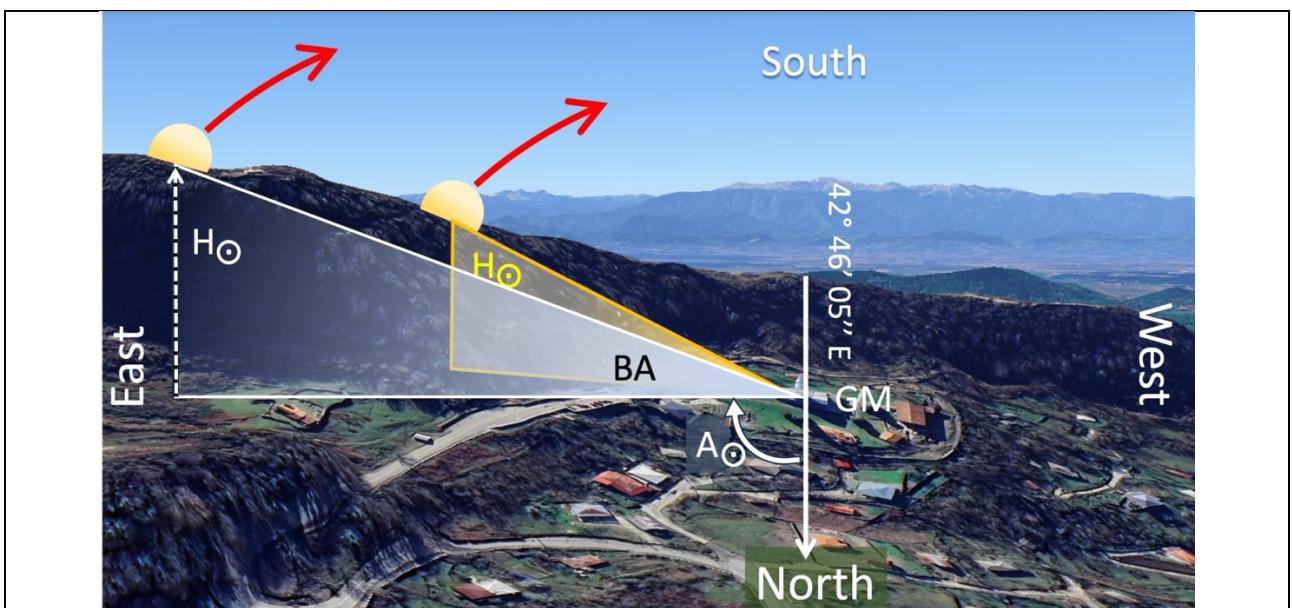


Fig.5. The blind angle (BA) after sunrise, when the Sun remains hidden behind the mountain relief and the Monastery is in the shadow.  $A_{\odot}$ : azimuth angle from the local meridian  $42^{\circ} 46' 05''$  E;  $H_{\odot}$ : solar elevation above the local horizon. The figure reports an example of two days, with the Sun emerging from the relief in different points. Every day is characterized by individual BA,  $A_{\odot}$  and  $H_{\odot}$  values.

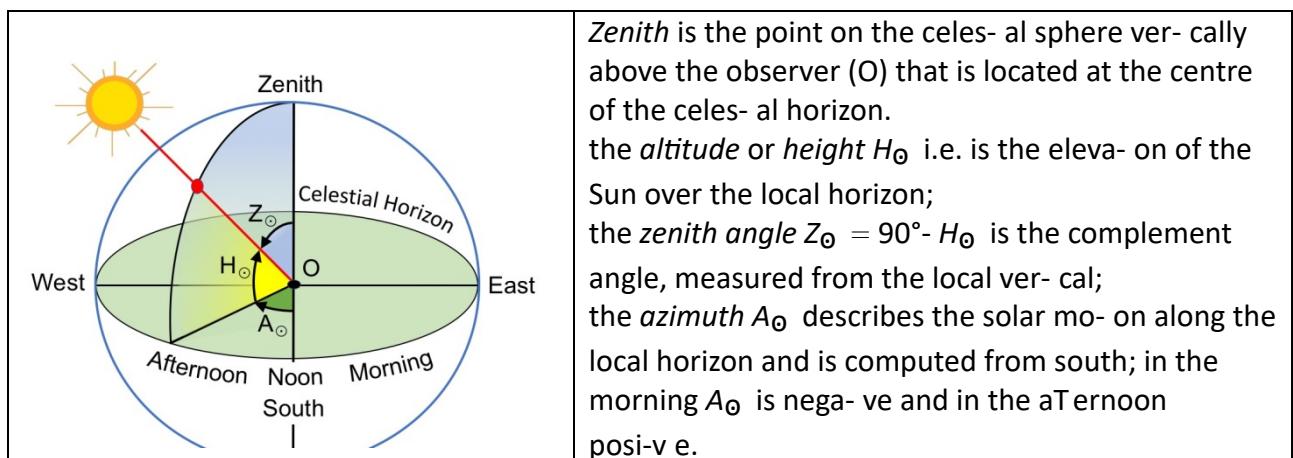
The potentially relevant azimuths lie between  $70^\circ$  and  $150^\circ$ , and BA between  $13^\circ$  and  $20^\circ$ . The most relevant combinations of Azimuth angles  $A_\odot$  blind angles BA are reported in Table 1.

**Table 1.** Azimuth angle  $A_\odot$  (computed from North) in which the Sun is hidden behind the mountain relief, when its elevation  $H_\odot$  over the local horizon is lower than the threshold given by the blind spot angle BA

Azimuth angle $A_\odot$ (degrees and decimals)	Blind angle BA (degrees and decimals)
70	13.5
80	14.8
90	17.6
100	19.4
110	19.8
120	19.8
130	19.8
140	19.8
150	19.0

The blind spot threshold has then been considered in the monthly calculations of the solar motions to highlight the shadowing period in the morning.

Solar coordinates and symbols are explained as follows:



### Calculation of the solar radiation on the Gelati Monastery

For the specific latitude  $\phi = 42^\circ 17' 42''$  crossing the Na-vity, and every day of the calendar year, defined by the solar declination  $\delta_\odot$ , the direct radiation can be calculated from the coordinates of the sun: the *altitude*  $H_\odot$  and the *azimuth*  $A_\odot$  i.e. the angular distance between the vertical circles containing the zenith and, respectively, the sun and the south point. Referring to south, the system is symmetric to noon, and the morning is characterized by negative azimuth values, i.e. degrees to culmination. Culmination is when the sun transits the local celestial meridian and reaches its highest altitude of the day, i.e. noon. At noon,  $A_\odot = 0$ . However, to be easily understandable to non-astronomers, the result is then transformed referring to north.

The astronomical formulae used to compute the solar coordinates are:

$$\sin H_\Theta = \sin \delta_\Theta \sin \phi + \cos \delta_\Theta \cos \phi \cos \tau \quad (1)$$

$$\sin A_\Theta = \frac{\cos \delta_\Theta \sin \tau}{\sqrt{1 - (\sin \delta_\Theta \sin \phi + \cos \delta_\Theta \cos \phi \cos \tau)^2}} \quad (2)$$

where the *hour angle*  $\tau = 180^\circ t/12$  is computed from the - me  $t$ , in hours and tenths of hour, from or to the culmina- on of the sun, i.e. from or to the true mid-day. This means that  $\tau$  is nega-ve in the morning, vanishes at noon and is posi- ve in the aTernoon.

The *solar declination*  $\delta_\Theta(j)$  for the  $j$ -th astronomical day is found in astronomical ephemerides tables or, for our purposes, it has been computed with the simple approxima- on

$$\delta_\Theta(j) = \delta_\Theta(0) \cos \frac{2\pi j}{365} \quad (3)$$

where the  $j$ -th day is computed aTer the winter sols- ce and  $\delta_\Theta(0) = -23^\circ 27'$  is the declina- on at winter sols-ce. The astronomical day  $j$  is then converted to the current day of the Gregorian calendar year by adding the 10 missing days from the winter sols- ce to 1<sup>st</sup> January.

The *flux density per unit time* or *intensity*  $I_p$  of the solar radia- on (also called *irradiation*) falling on horizontal, ver- cal or arbitrarily inclined planes facing the direc- on  $A_p$  (computed referring to the astronomic south, which represents the plane of the true noon at culmina- on) and inclined by the angle  $\beta$  with the plane of the local horizon has been computed with the formula

$$\begin{aligned} I_p = I_0 & \{ \cos \beta (\sin \delta_\Theta \sin \phi + \cos \delta_\Theta \cos \phi \cos \tau) + \\ & + \sin \beta \{ \cos A_p [\tan \phi (\sin \delta_\Theta \sin \phi + \cos \delta_\Theta \cos \phi \cos \tau) - \sin \delta_\Theta \sec \phi] + \\ & + \sin A_p \cos \delta_\Theta \sin \tau \} \} \end{aligned} \quad (4)$$

where  $I_0$  is the intensity of the solar beam (near the surface) and the irradiation on a horizontal or vertical plane is obtained by setting  $\beta = 0^\circ$  or  $\beta = 90^\circ$ , respectively.

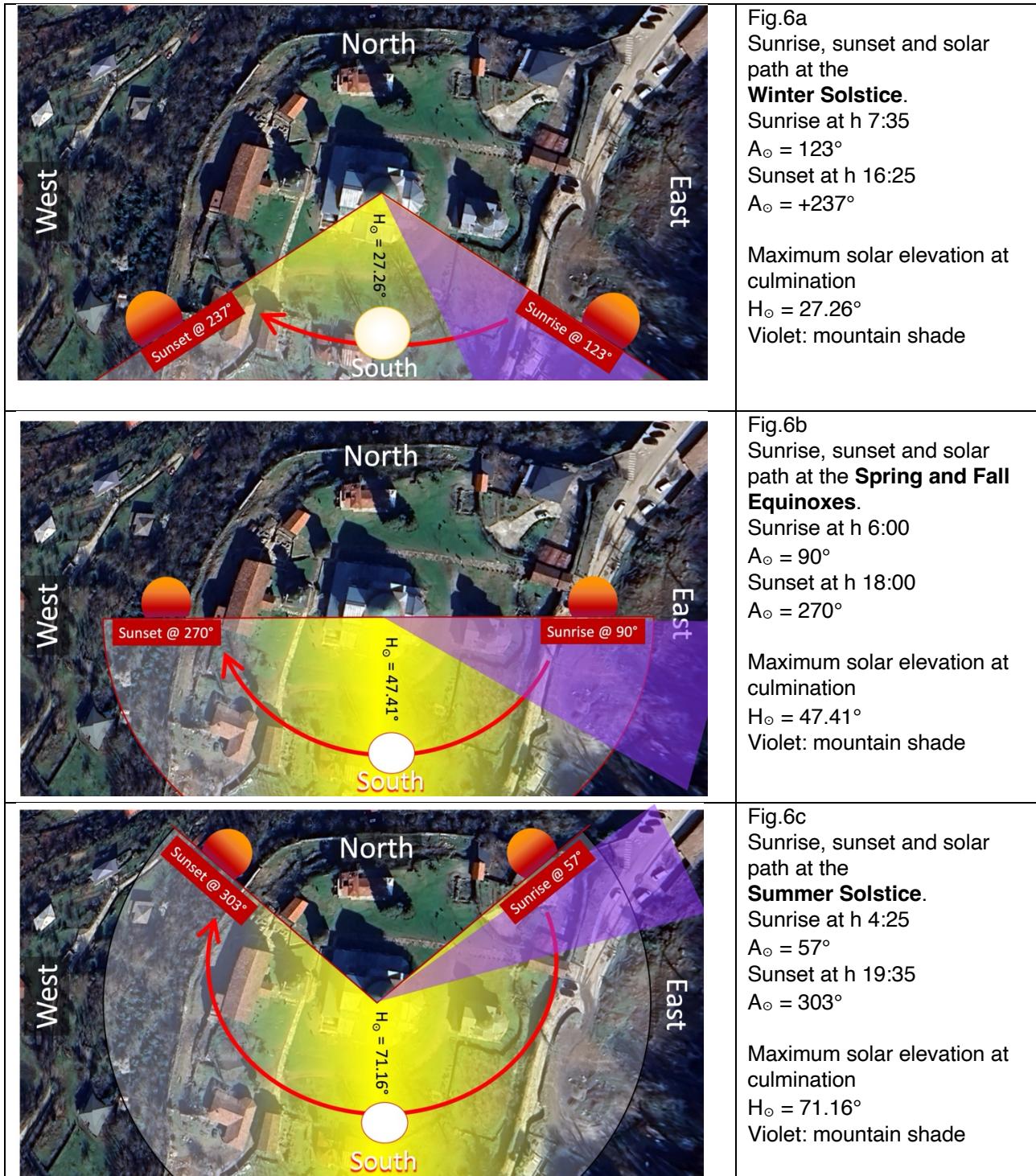
Finally, it has been considered that the solar radiation is attenuated passing through the atmosphere, depending on the actual atmospheric thickness crossed by the solar beams, which depends on the solar altitude and the incoming wavelength  $\lambda$  (Bouguer–Lambert law), i.e.

$$I(\lambda) = I_0 \exp(-\alpha(\lambda) m) \quad (5)$$

where  $I_0$  is the intensity of incident radiation (outside our atmosphere),  $I(\lambda)$  is the intensity after having crossed the optical air mass  $m$ , and  $\alpha(\lambda)$  is a coefficient to reproduce the absorption.

## PART II: SYNTHESIS AT THE TWO SOLISTICES (THE TWO EXTREMES) AND THE TWO EQUINOXES (THE MIDWAY SITUATION)

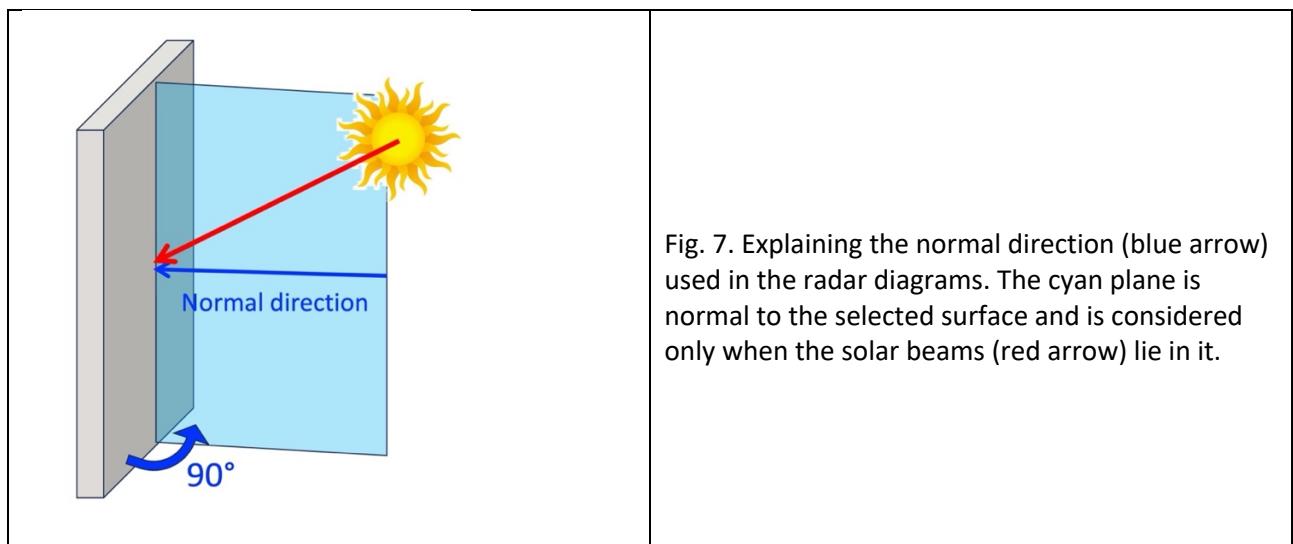
The - me (in hours h, true astronomic - me) and the azimuth angle  $A_{\odot}$  of sunrise and sunset in degrees from North, as well as the highest elevation of the Sun  $H_{\odot}$  over the local horizon at culmination, are calculated for the Winter Solstice, Equinoxes and Summer Solstice (Fig.6)



The estimated blind spot, where the Sun is low and hidden behind the mountain's relief, is highlighted in purple. Since the local topography is irregular, this should be considered a first approximation (i.e., more qualitative than quantitative) to facilitate understanding.

### The solar energy impinging on the external walls (facades and tambour) of the Nativity Church

The next pages include two diagrams. These can be interpreted considering the situation represented in Fig. 7. Let us consider a selected surface (grey). The directions considered in the radar diagram are normal to the selected surface (i.e. the blue arrow). The 'normal direction' lies in the normal plane (cyan, forming a 90° angle with the surface) which includes the solar beam (red arrow) impinging on the surface.



**Instantaneous flux of solar energy Fig.8a.** The upper graph represents the instantaneous flux of solar energy ( $\text{kw m}^{-2}$ ) impinging per second and square meter on the four vertical walls (the facades) of the Nativity Church, as indicated.

The time in abscissa is the true astronomic time (or local solar time, i.e. reference is made to the culmination, when the Sun crosses the local meridian 42°46' 05" East. Formally, there is a time shift between the True Astronomical Time (where culmination is at 12:00) and the official clock time in Georgia, but this does not affect the calculated amounts of the incoming solar radiation.

The model simulations suppose a free horizon around the building, without obstacles or surrounding mountains. However, depending on the season, the mountain relief on the eastern side shields the early morning radiation from about one to two hours after sunrise, until the Sun rises above the ridge. In the upper energy flux diagrams, the night-time is highlighted pale blue, and the approximated period of mountain shadow orange. It must be specified that the slope is uneven in level and distance, so that the evaluation of the projected shade should be considered a rough estimate.

**Daily total income of solar energy (Fig.8b).** The lower radar diagram represents the total income of solar energy ( $\text{kJ m}^{-2}$ ) that has been supplied during the whole day per square meter on the horizontal plane (green) and on vertical surface (red) of the tambour, with detail for any selected direction (10° resolution). Of course, this can be applied to any other vertical surface.

The interpretation of the radar diagram requires a further comment. The 0-360° directions make sense for vertical surfaces, and every direction is determined by the incoming solar beams as explained.

In this diagram, the mountain shadow has not been reported, but lies for some 20°-30° on the eastern side, starting from sunrise, when the Sun is low over the horizon and is masked behind the mountain ridge. To get a more precise idea, the reader can obscure the first two or three 10° partitions after sunrise, where the red curve on the right side begins.

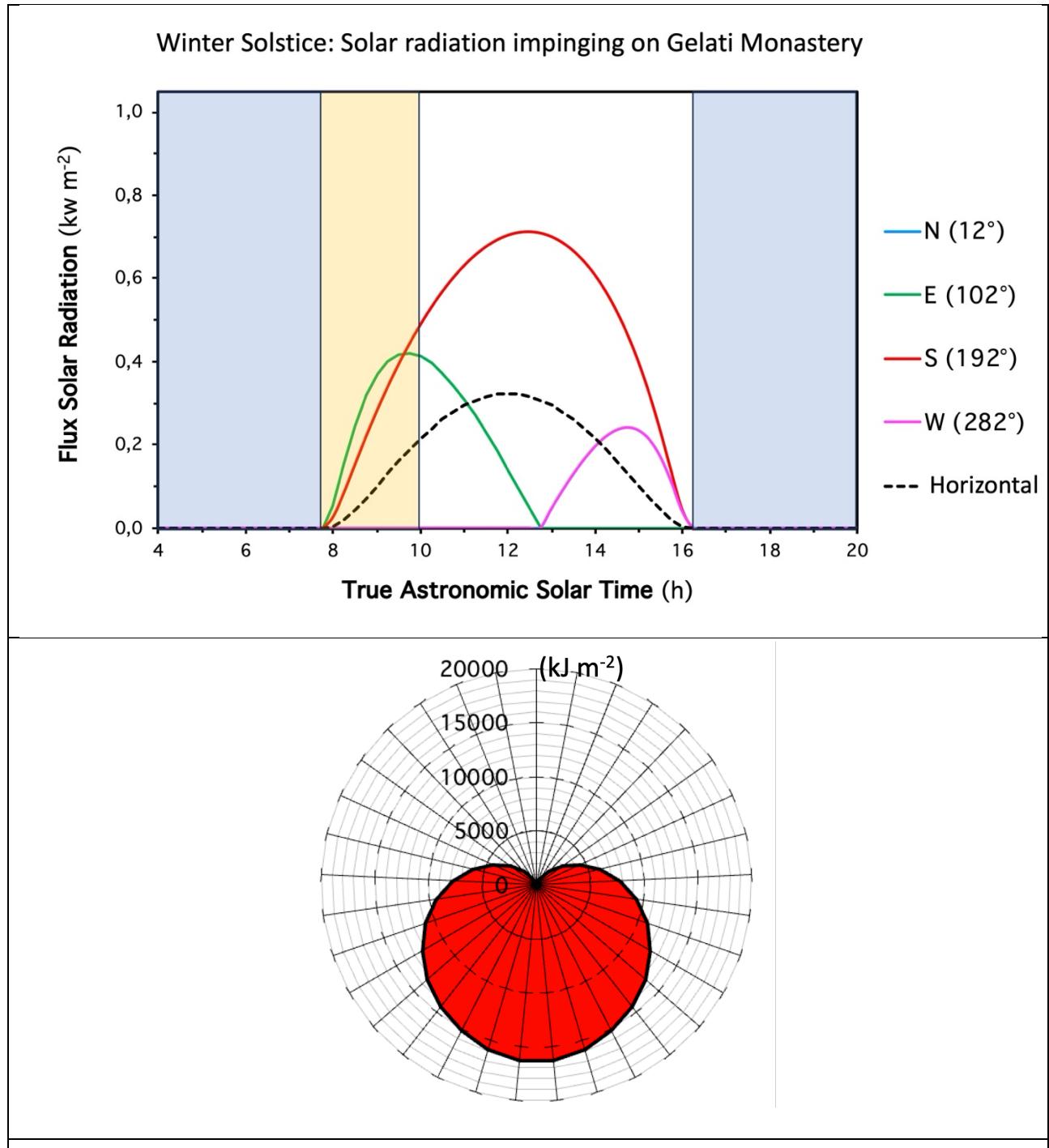
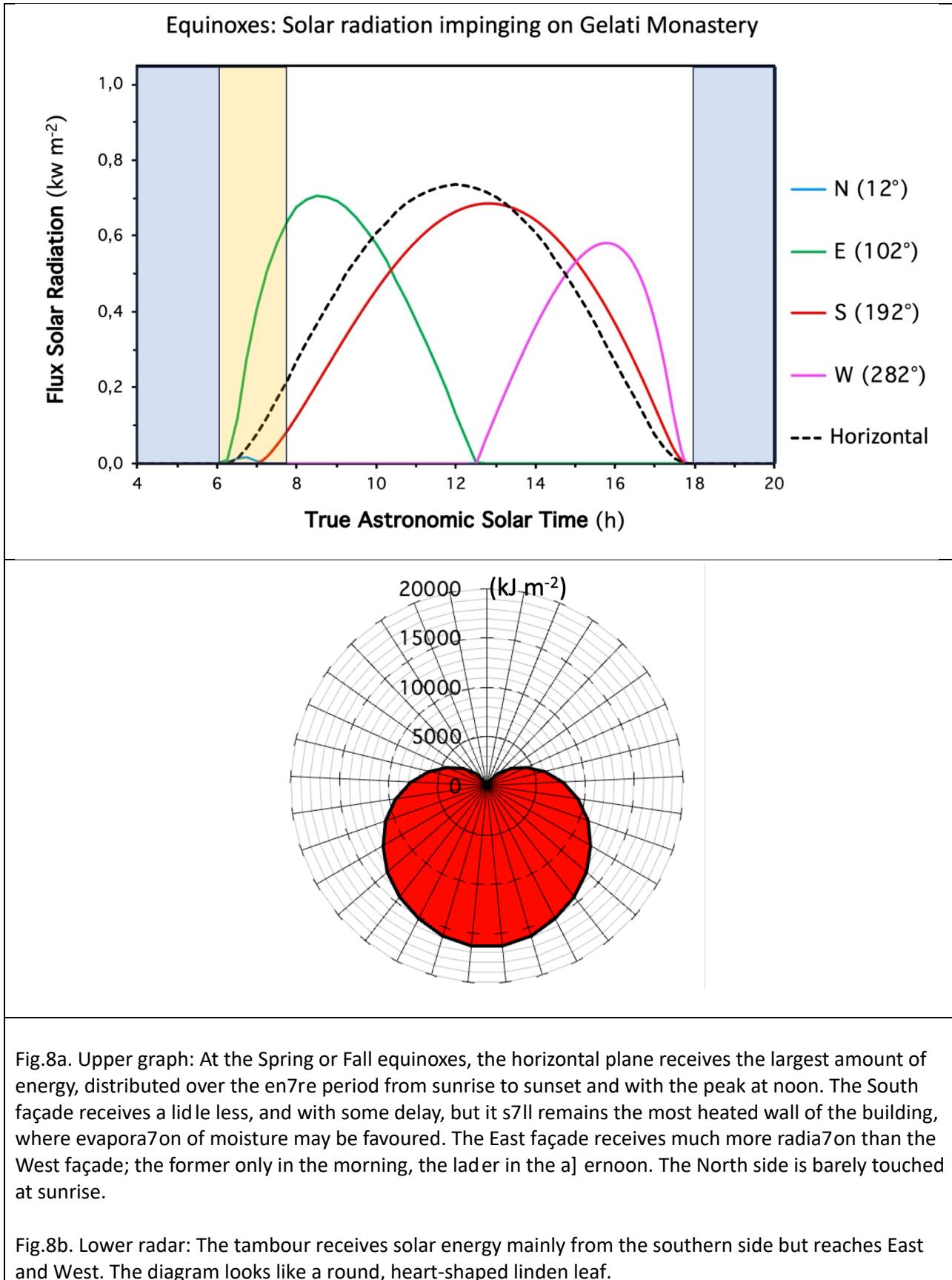
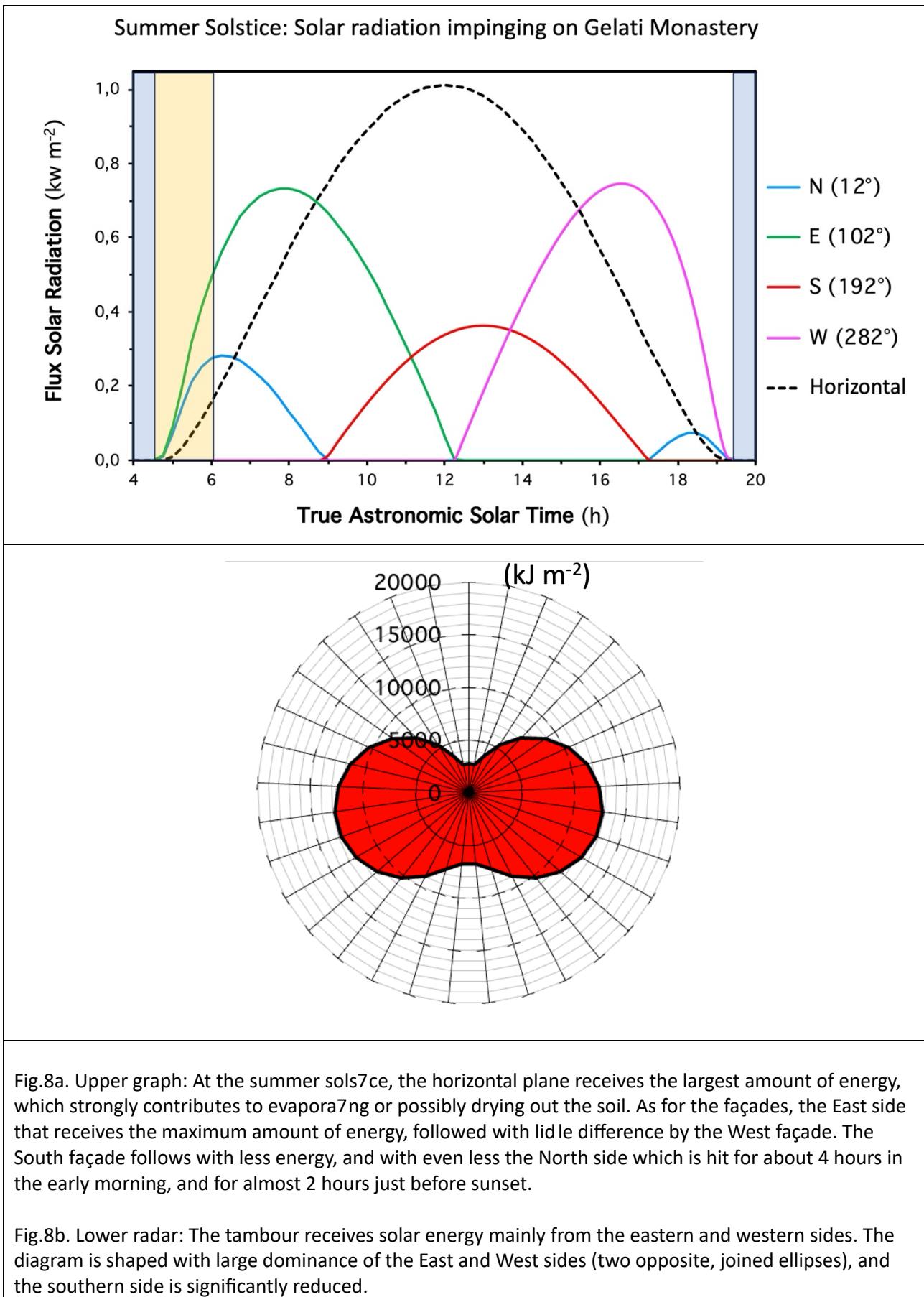


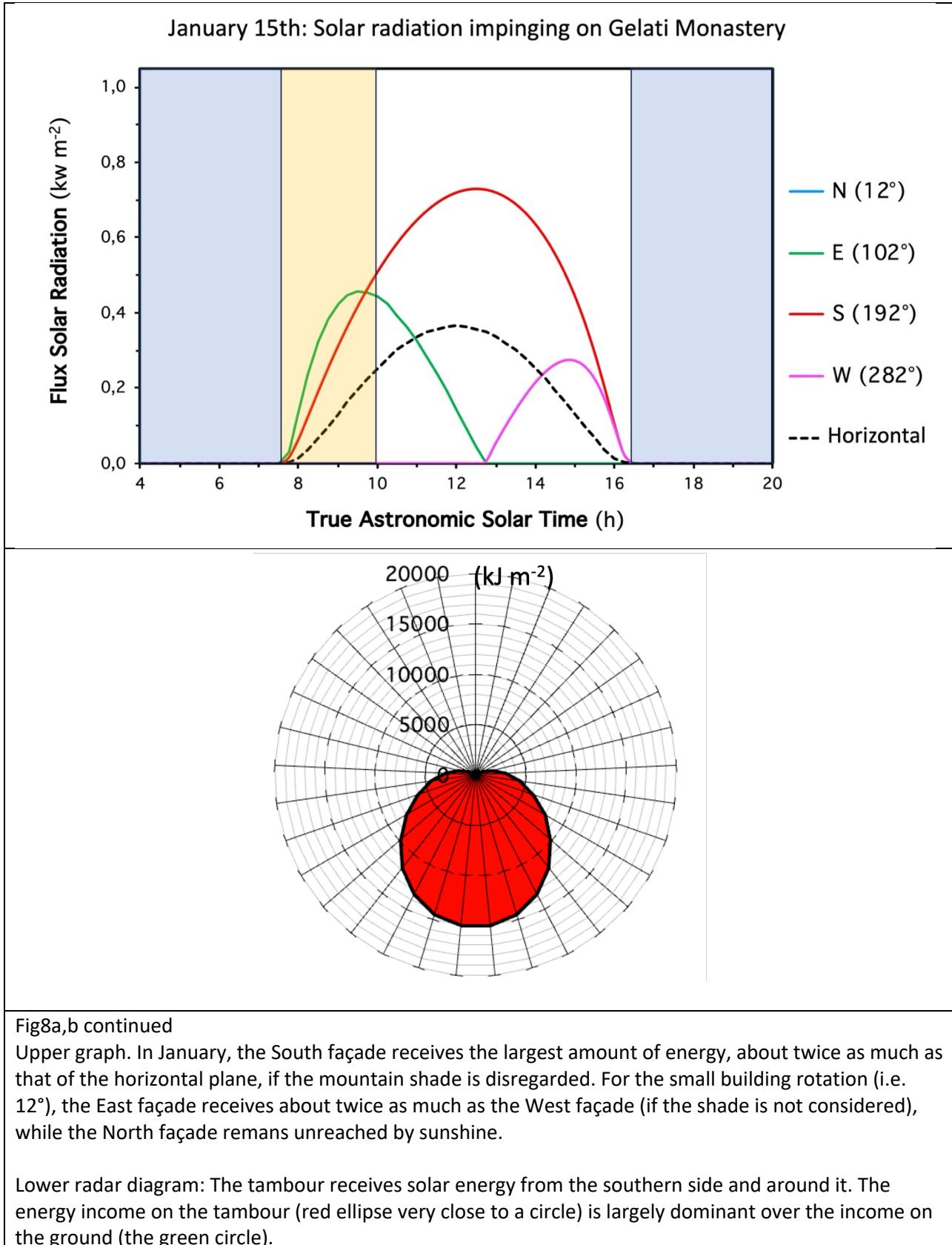
Fig.8a. Upper graph: At the winter solstice, the South façade receives the largest amount of energy, about twice as much as that of the horizontal plane. A consistent fraction of this flux is stopped by the mountain shade. For the small building rotation (i.e. 12°), the East façade receives about twice as much as the West façade (if the shade is neglected), while the North façade remains unreach by sunshine.

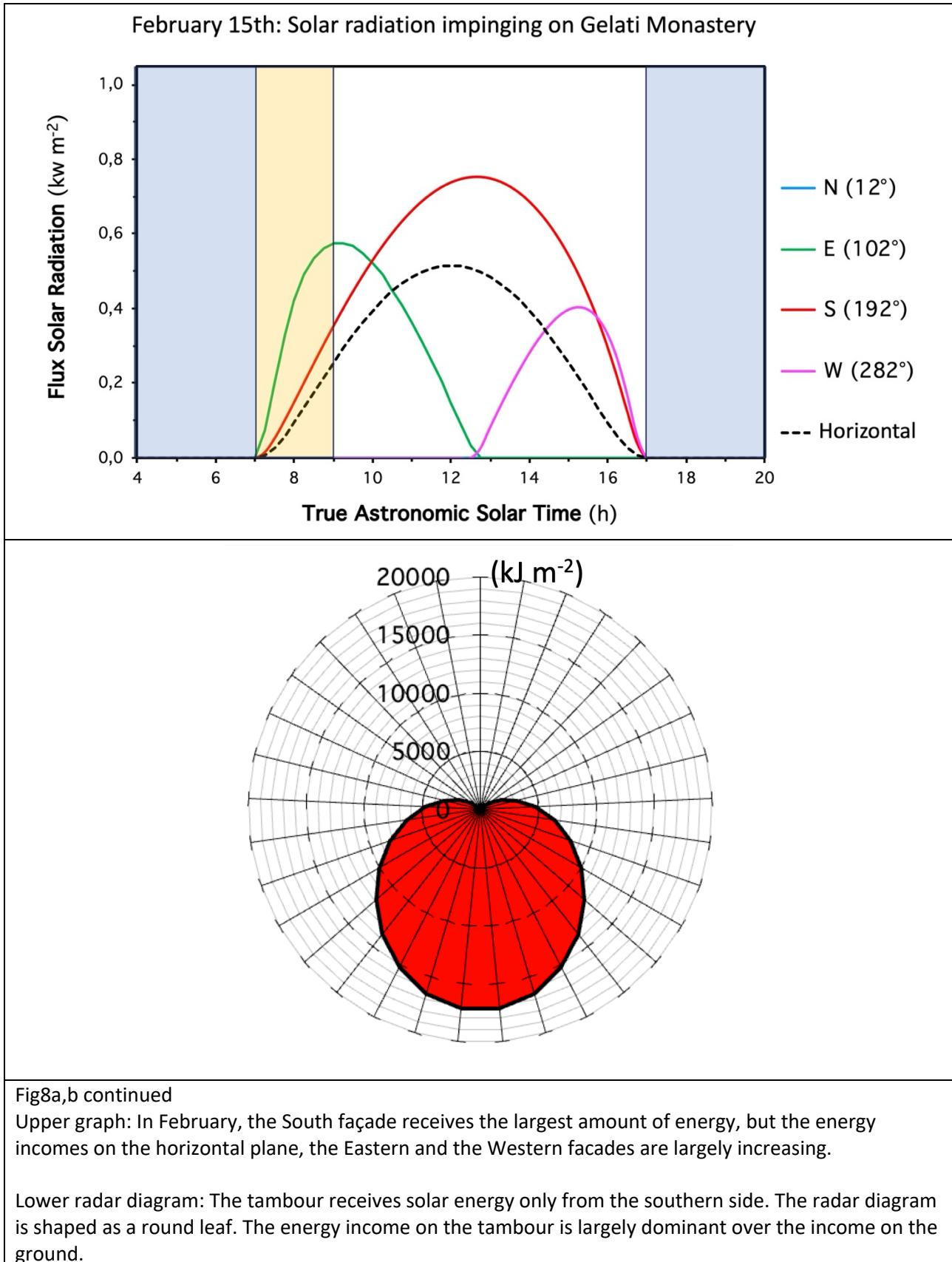
Fig.8b. Lower radar diagram: The tambour receives solar energy especially from the southern side, and around it (red ellipse).





### PART III: MONTLY VALUES (Fig8a,b continued)





March 15th: Solar radiation impinging on Gelati Monastery

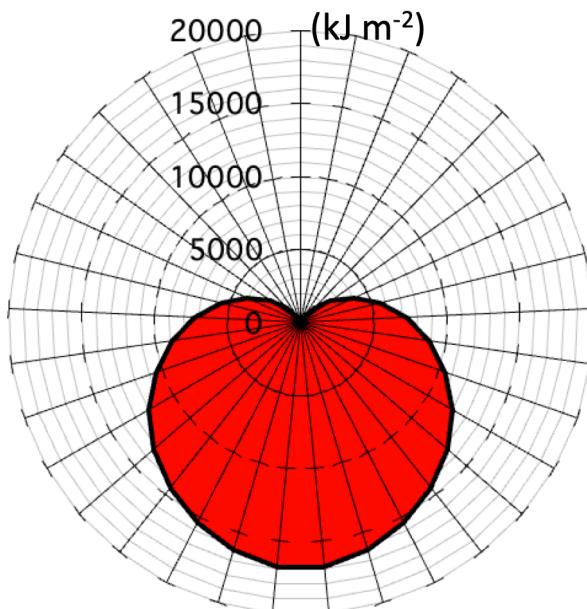
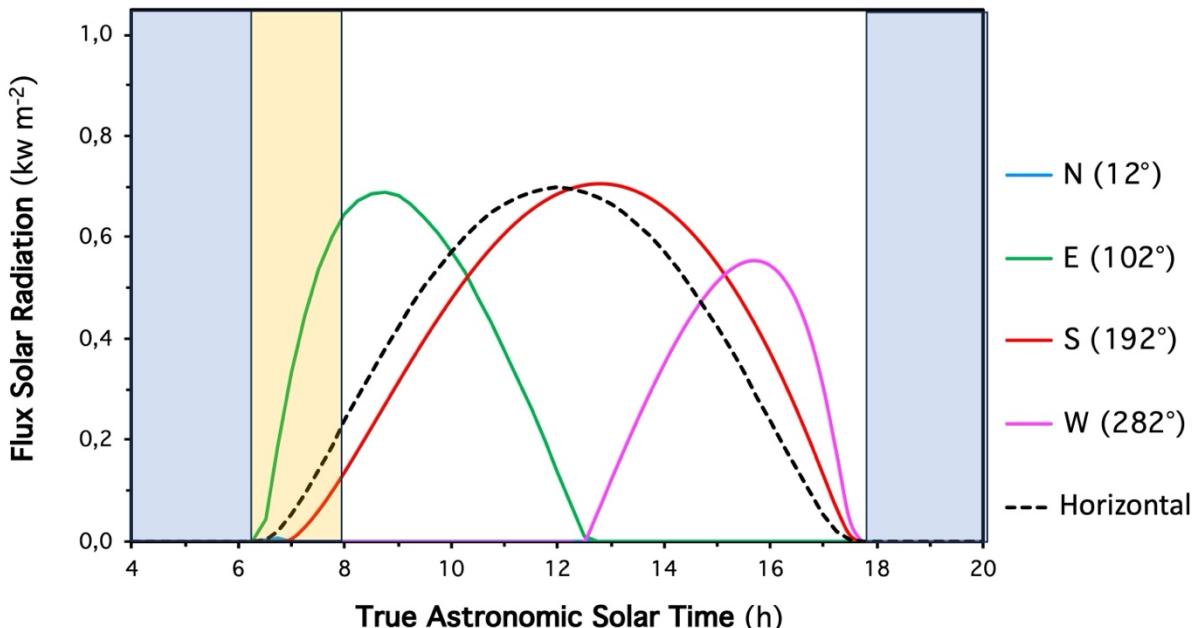


Fig8a,b continued

Upper graph: At mid-March the situation is similar to the Spring Equinox. The horizontal plane has reached the South line, and the solar contribution is almost the same on both. The East facade receives less energy than South, but much more than the West facade; the former only in the morning, the latter in the afternoon. The North side is barely reached by grazing beams at sunrise.

Lower radar diagram: The tambour receives solar energy mainly from the southern side. The diagram is heart-shaped like a linden leaf and reaches East and West.

April 15th: Solar radiation impinging on Gelati Monastery

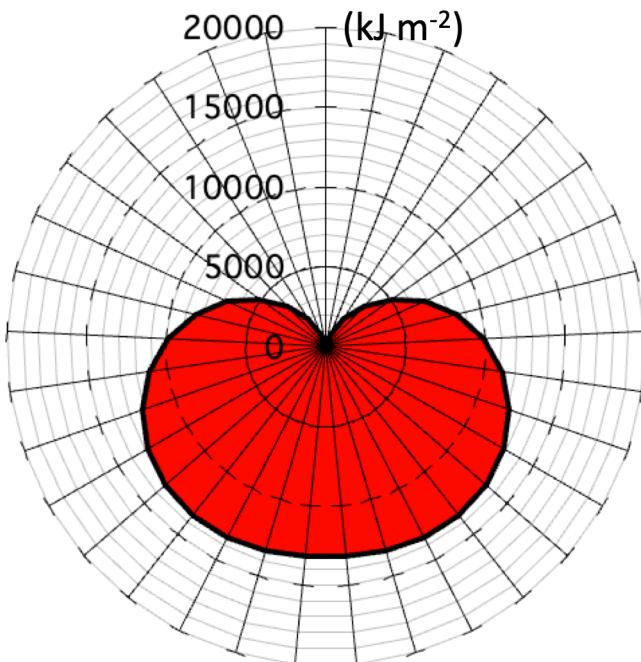
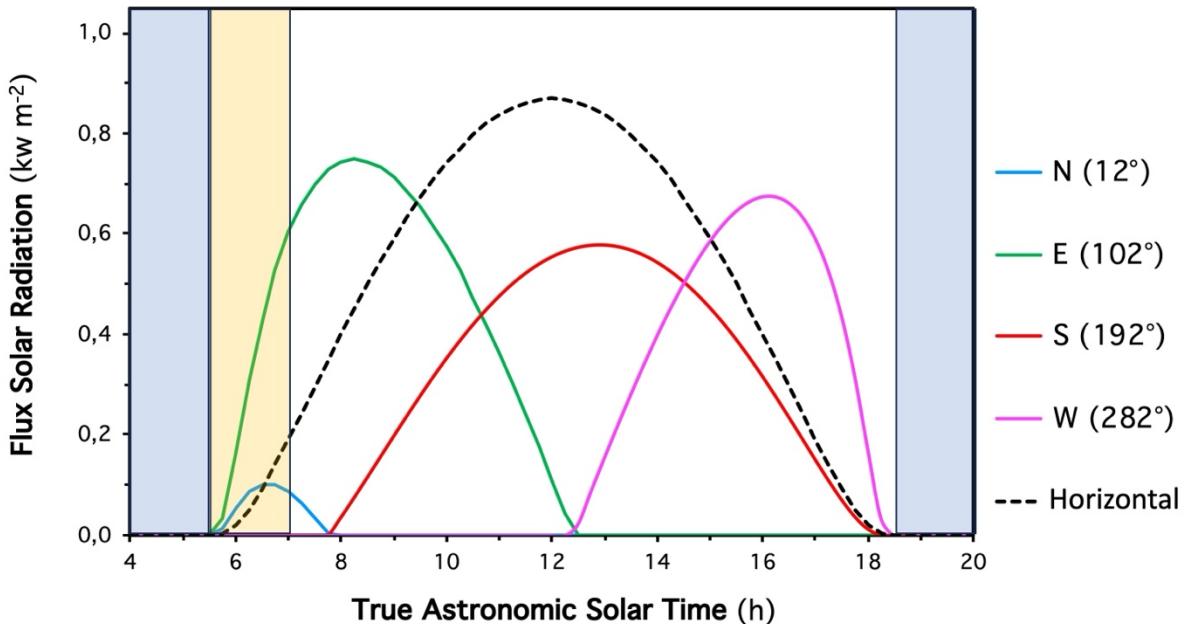
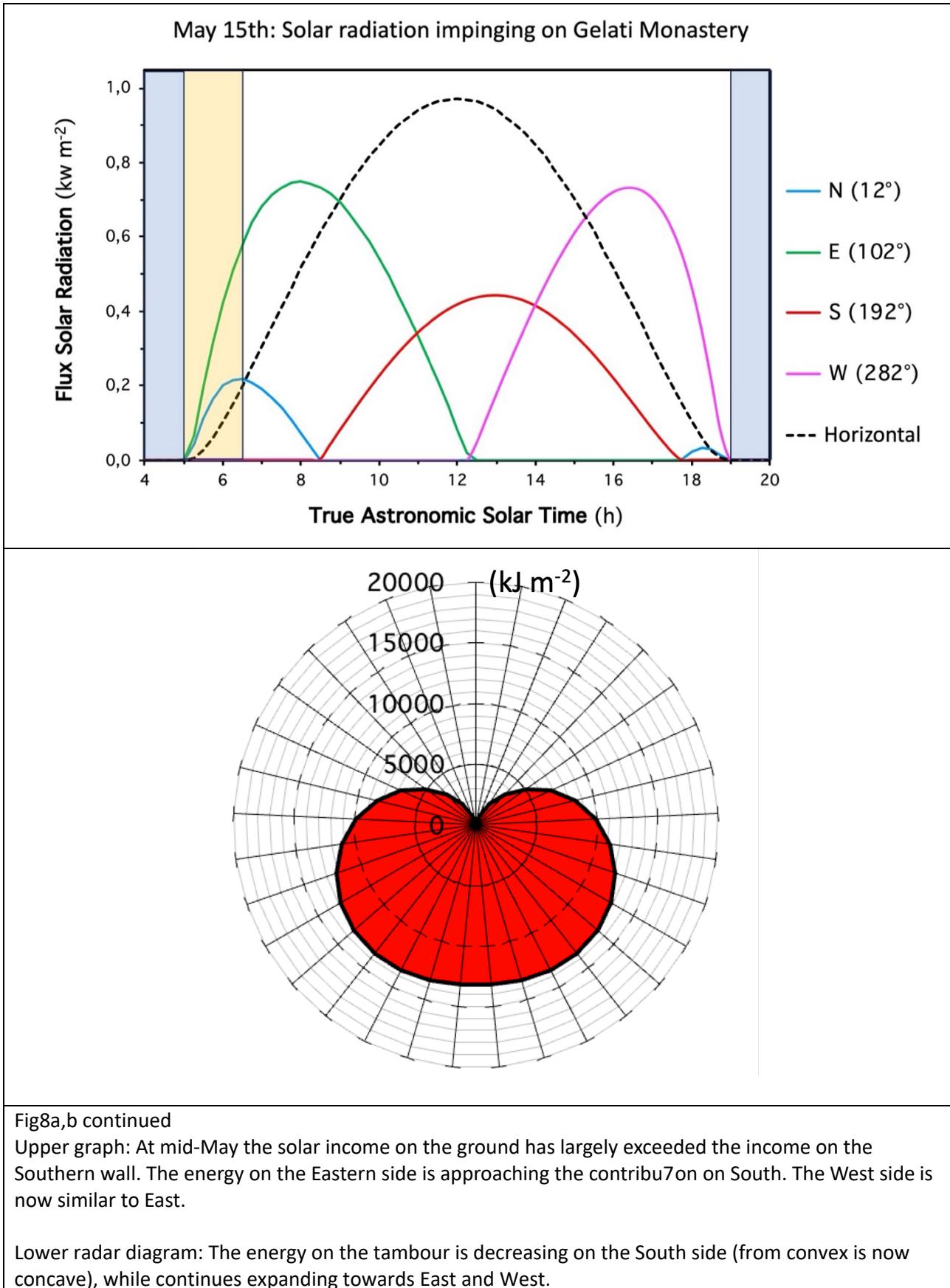
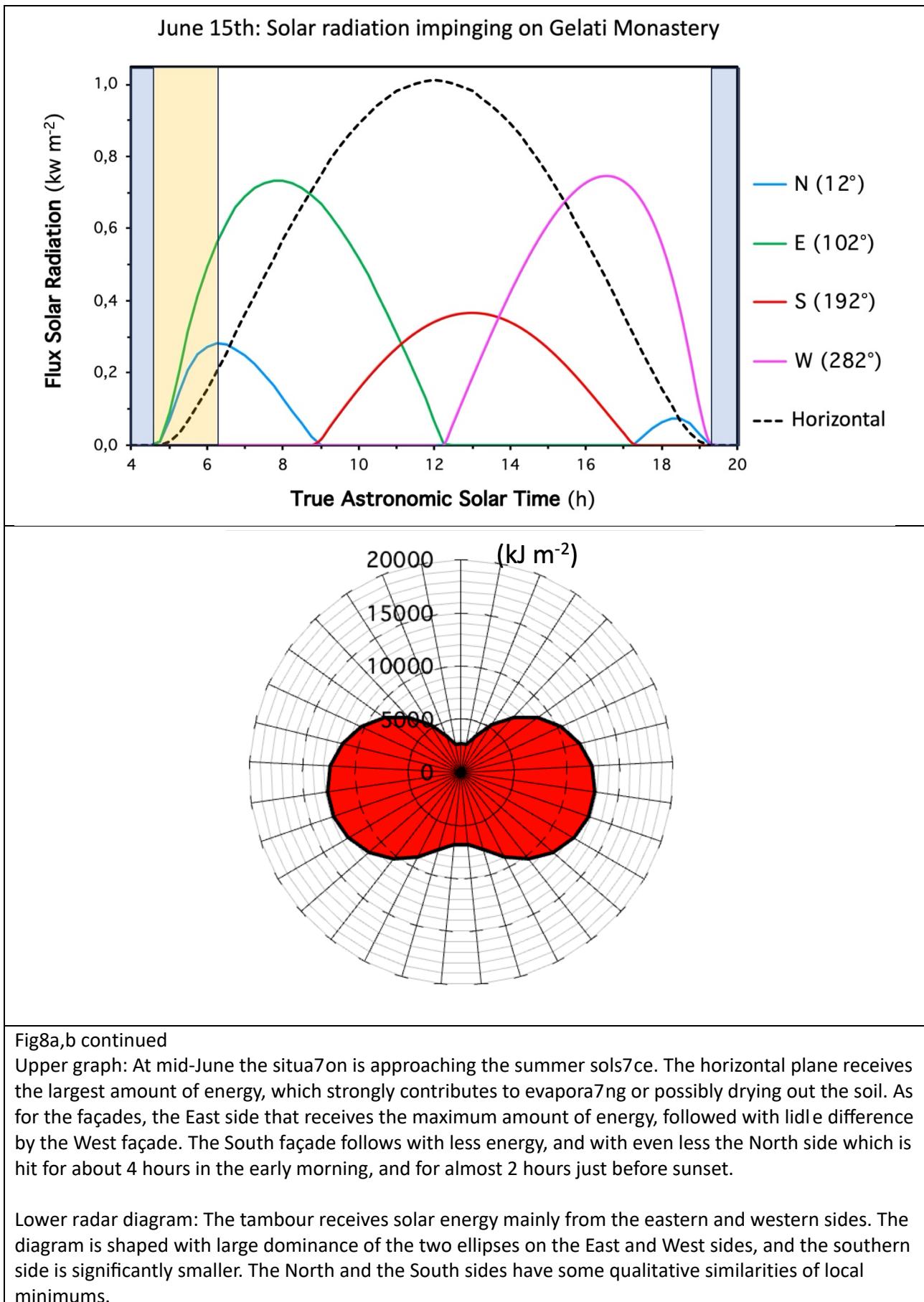


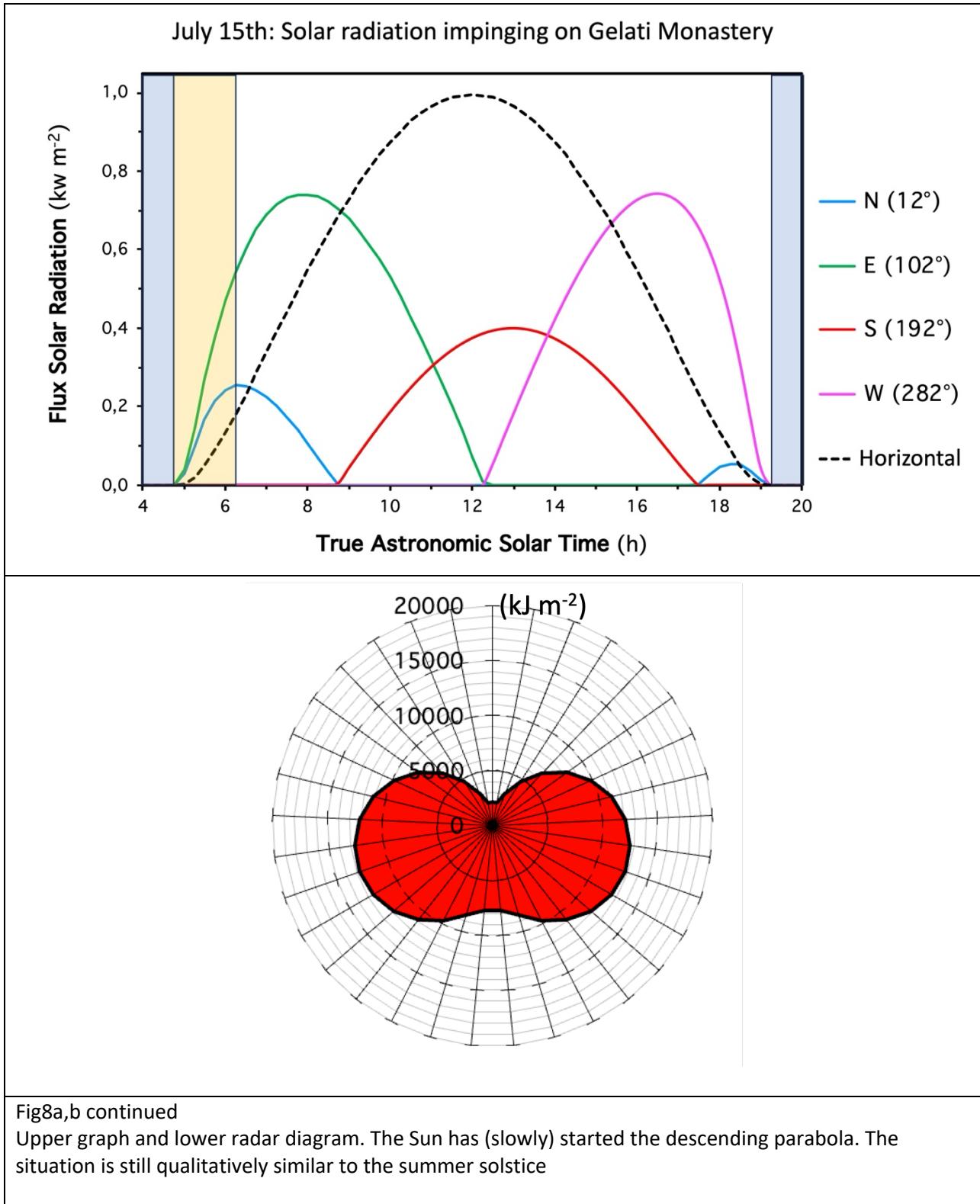
Fig8a,b continued

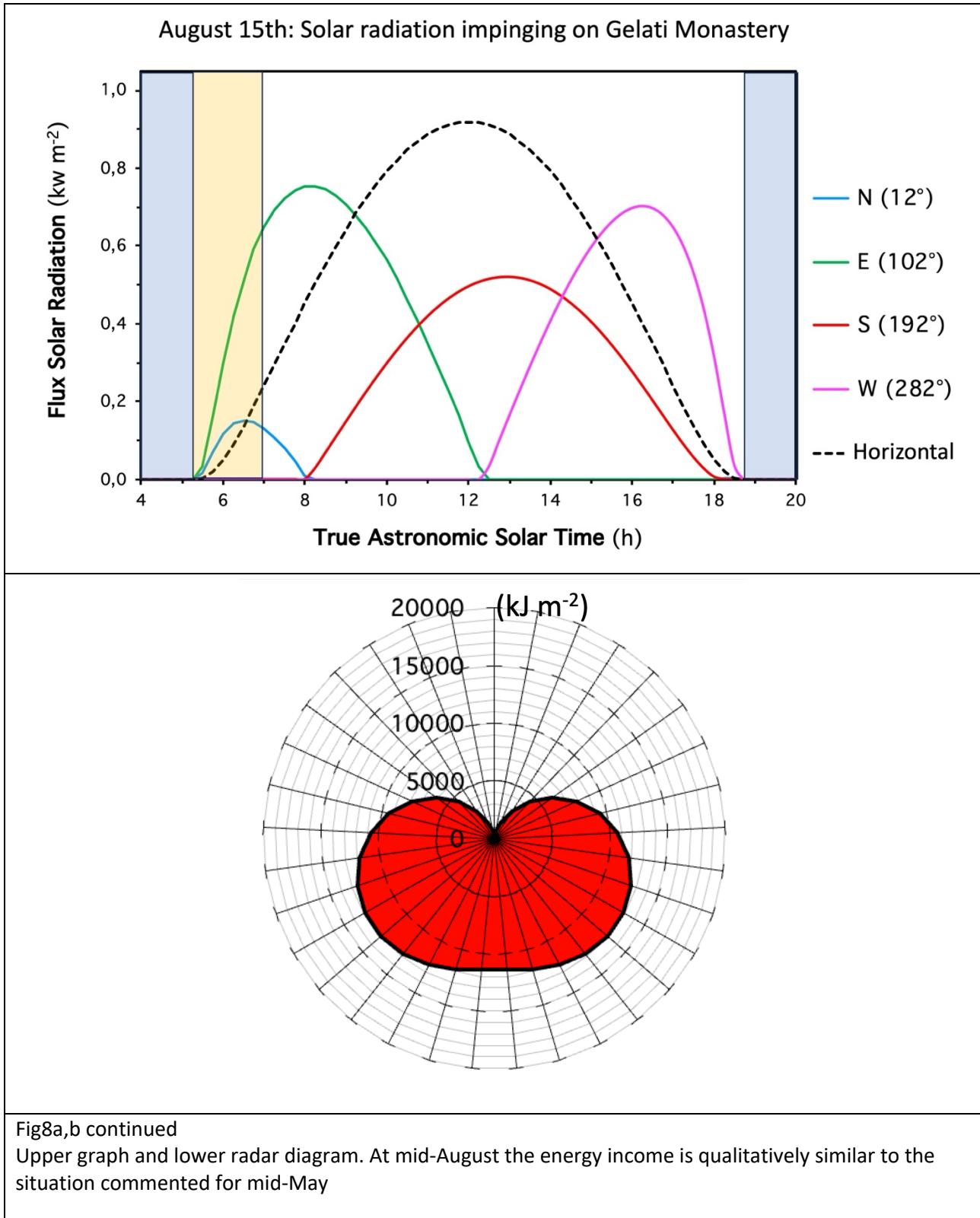
Upper graph: At mid-April the solar income on the ground has largely exceeded the income on the Southern wall. The energy on the Eastern side is approaching the contribution on South. The West side has decidedly less. The North side starts to appear with grazing beams.

Lower radar diagram: The energy on the tambour is decreasing on the South side, while expanding towards the East and West, like a cross-section of a tomato.









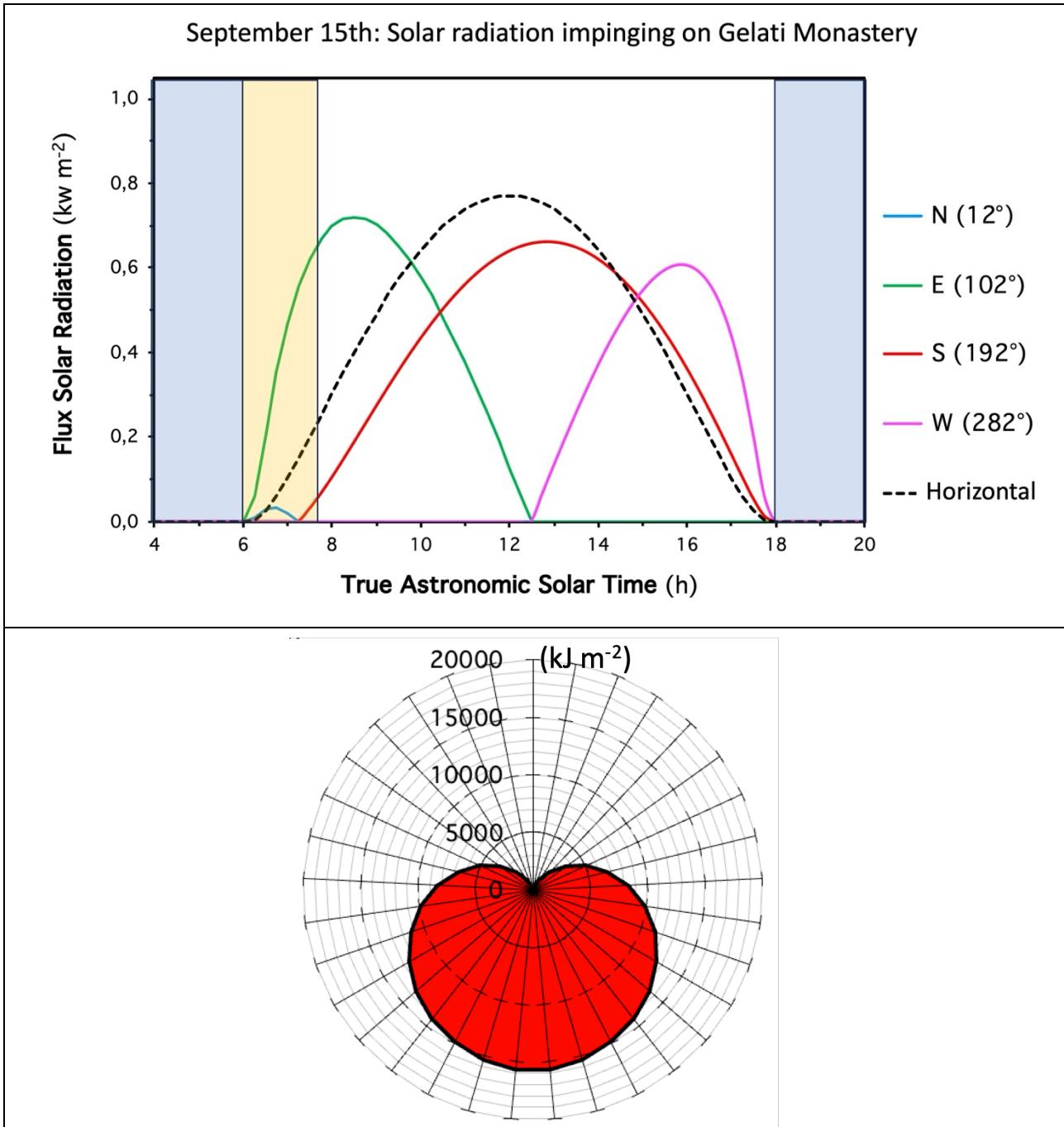


Fig8a,b continued

Upper graph: At mid-September the situation is like the Fall Equinox. The horizontal plane receives the largest amount of energy, distributed over the entire period from sunrise to sunset and with the peak at noon. The South facade receives a little less, and with a certain time lag, but it still remains the most heated wall of the building, where evaporation of moisture may be favoured. The East facade receives much more radiation than the West facade; the former only in the morning, the latter in the afternoon. The North side is barely touched at sunrise.

Lower radar diagram: The tambour receives solar energy mainly from the southern side. The diagram is heart shaped and reaches East and West.

October 15th: Solar radiation impinging on Gelati Monastery

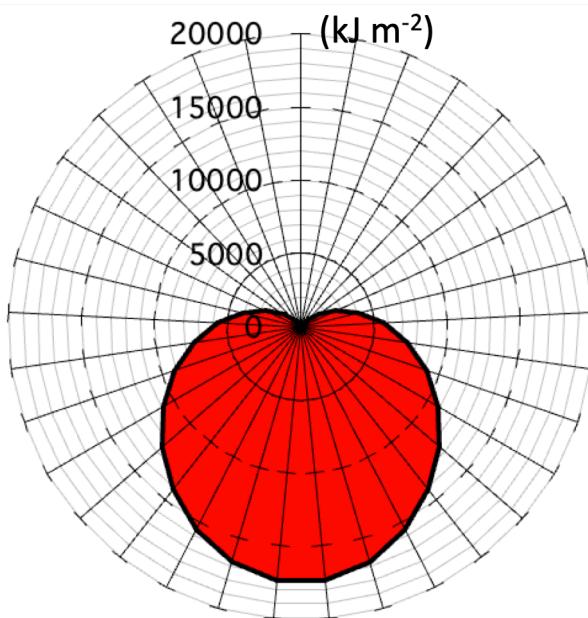
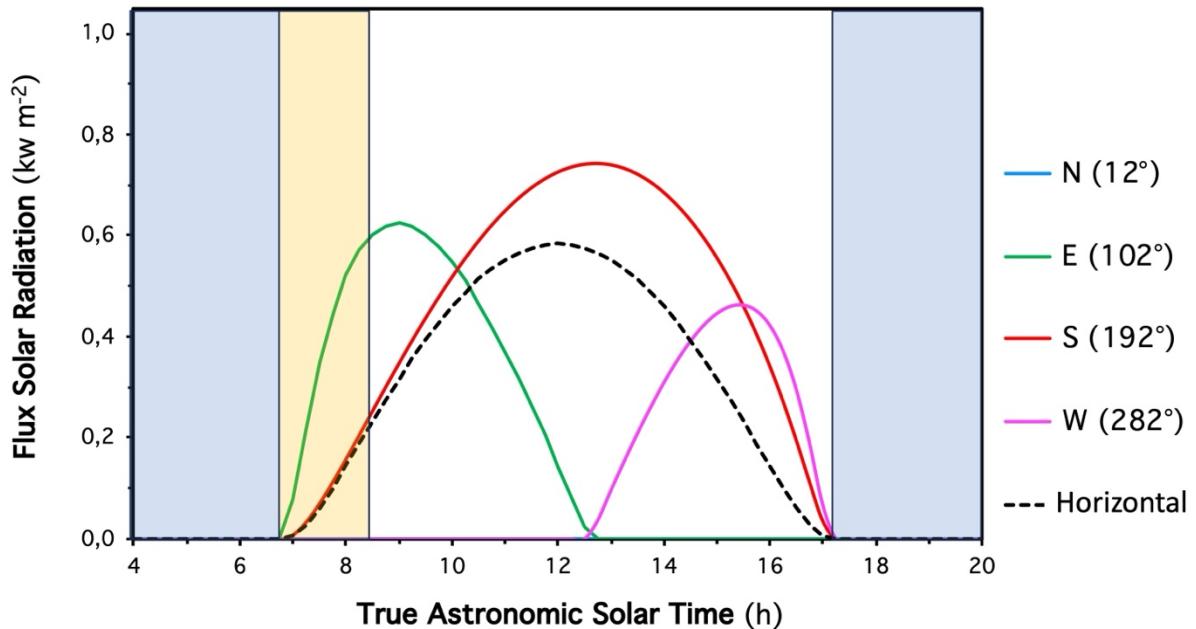


Fig8a,b continued

Upper graph and lower radar diagram. At mid-October the energy income is qualitatively similar to the situation commented for mid-February

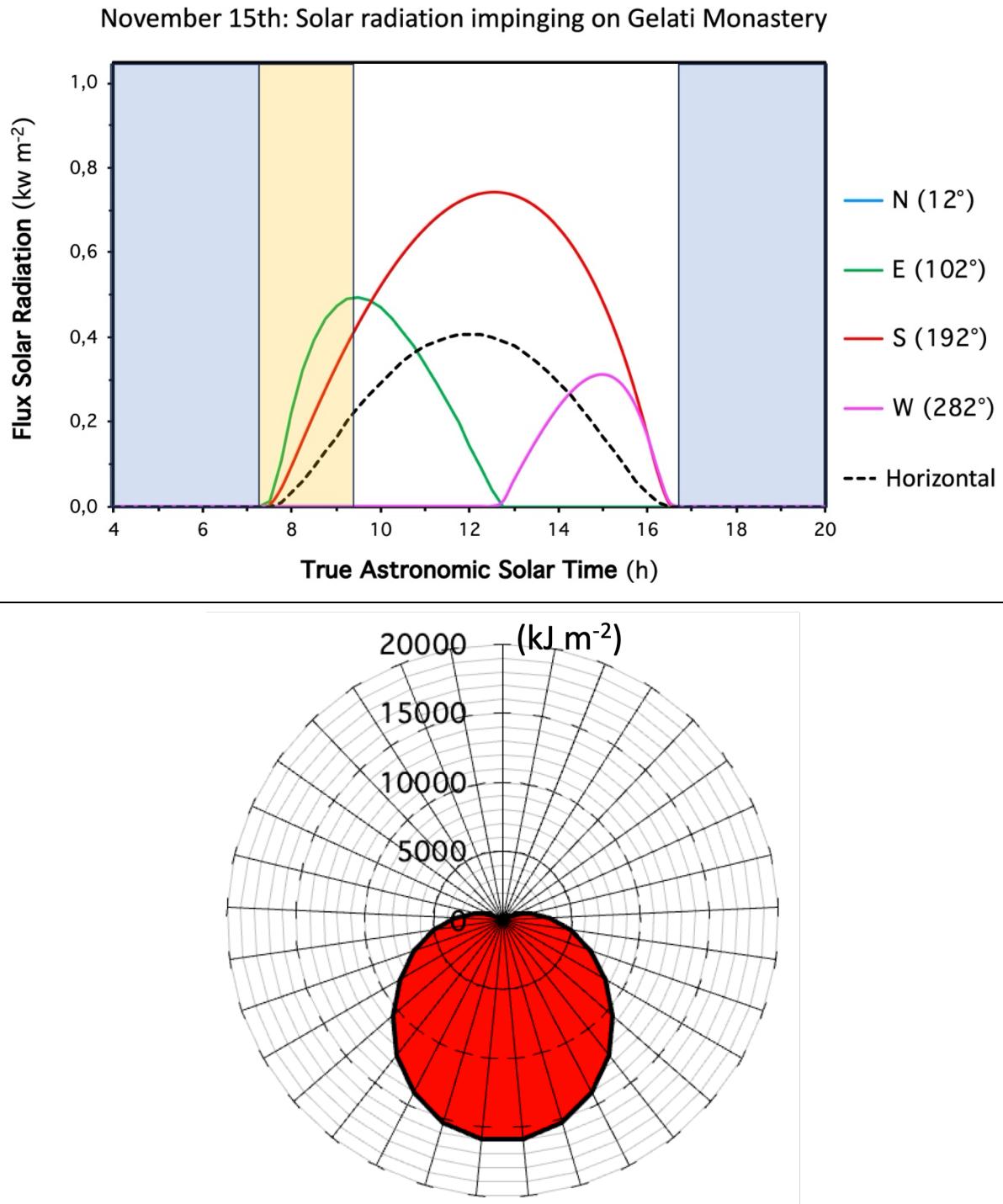
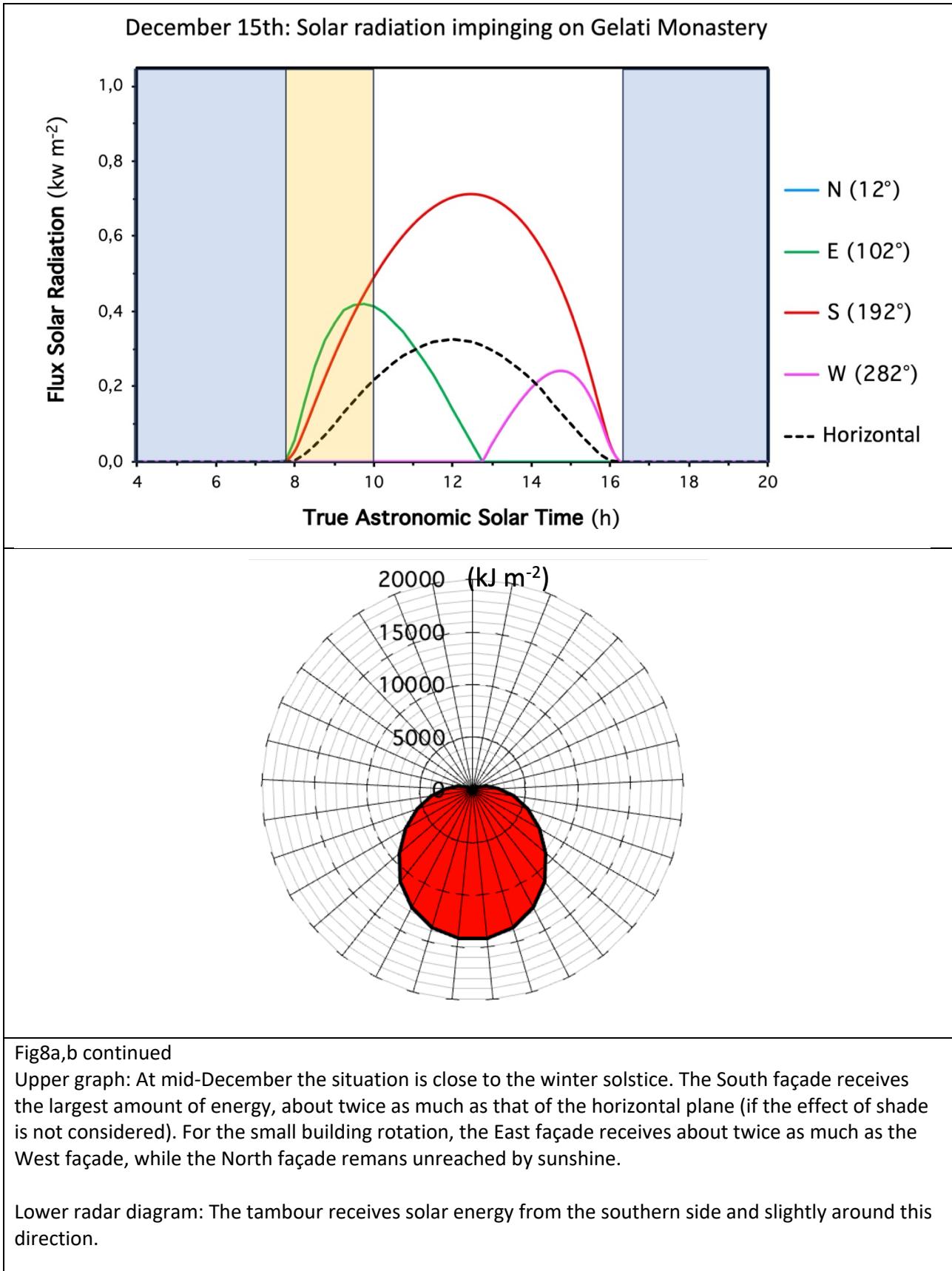


Fig8a,b continued

Upper graph and lower radar diagram. At mid-November the energy income is qualitatively similar to the situation commented for mid-January



## PART IV. PRACTICAL CONSIDERATIONS

### The alignment departing by 10° from East-West

The alignment of the three Churches, i.e., St George, Na- vity of the Virgin Mary, and St Nicholas, is not oriented East – West as usual, but with a clockwise departure of around 10°, as shown in Fi.2. Consequently, the buildings are aligned with the sunrise and sunset on the astronomic horizon passing through Gela- on March 3<sup>rd</sup> and October 9<sup>th</sup>.

A possible hypothesis is that the architect intended to align the Churches of the Monastery with the Sun coming out from behind the mountain ridge on the Na- vity Day of the Virgin Mary, i.e., September 8<sup>th</sup>. The architect had to wait long enough for the Sun to rise from behind the ridge; then he oriented the churches to the true direc- on from which the Sun emerged.

According to the imagery and mapping of Google Earth, Gela- lies at some 400 m eleva- on above the average sea level, and in the eastern side is surrounded by a mountain ridge which includes the St Nino Father Monastery and the Upper Gela- Church of Icon of the Most Holy Mother of God. The level of this relief is some 100 m above the Church of the Na- vity of Virgin Mary, which sees the relief at an angle of some 18°-20° above it, on the eastern side.

This requires calcula- ng the mo- on of the Sun as perceived by Gela- on September 8<sup>th</sup>. Referring to a horizontal plane through the Church of Na- vity, Gela- , the sunset (h 6:14 regular -me in Georgia, or h 5:40 true astronomic - me) occurs in the direc- on 84° measured clockwise star- ng from North (i.e. 6 ° before East). About 100 minutes later (around 8 a.m. Georgian - me) the Sun has reached the direc-on 100° (i.e. 10° aTer East), is aligned with the three Churches in Gela- , and has reached an eleva- on of 16° degrees above the astronomic horizon. At that moment, the solar beams should be grazing the mountain ridge, making sparkling the trees there. In a short, the upper border of the Solar disk should emerge from behind the mountain ridge and start to illuminate the Church of the Na- vity and the whole Gela- courtyard. On September 8<sup>th</sup>, the feast of the Na- vity of the Virgin, the Sun should come out from behind the ridge, in a posi- on aligned with the three churches of the Gela- Monastery (Fig.9).



Fig.9. Alignment of the Church of the Na- vity with the solar disk emerging from the mountains on September 14, 2025, Gregorian Calendar (Photo Father Kirion)

The above hypothesis is supported by the astronomical calculations, but needs practical confirmation on site. It should be considered, however, that the Monastery was founded in 1106, with the Julian calendar. When Pope Gregory XIII introduced the Gregorian calendar in 1582, he omitted 10 days from the calendar to realign it with the equinoxes. Therefore, in 1006 the difference was 6 days, i.e. the Julian date September 8<sup>th</sup> corresponded to the Gregorian date September 14<sup>th</sup>. On September 14<sup>th</sup> 2025, after some overcast days, the weather improved and Father Kirion could take a picture with a drone (Fig.9). The alignment between the Church and the Sun is rather good, which strengthens the hypothesis of the dedication to the Virgin. Looking from ground level, the Sun takes a few minutes longer to emerge from behind the ridge, slightly shifted to the right and in better alignment.

### **Considerations on the energy supply**

In the early morning, the shadow cast by this mountain relief delays the arrival of the direct solar beams by about one to two hours, depending on the season.

It has been seen that as the seasons change, not only does the amount of solar energy that falls on each wall change, but also that the relationships between individual walls.

The North wall is the one that receives the lowest quantity of solar energy, while the surfaces that receive the highest amount are, depending on the season, the South facade, the Western and Eastern facades or the horizontal plane.

The vertical surfaces as a whole collect much more solar energy than the horizontal plane, and this is particularly evident in the tambour (which includes all directions) where the red area representing the energy collected throughout the day by the ensemble of vertical surfaces is always largely dominant with respect to the green circle, representing the horizontal plane.

Over the year, each wall collects a certain amount of solar energy, which will raise the external surface temperature of the walls, establish a thermal gradient inside the masonry, and regulate the temperature of the internal counter-facade (although much less than the external surface).

The thermal gradient inside the masonry is responsible not only for temperature, but also has practical effects on the migration of the moisture penetrated because of rainfall or condensation.

It should be remembered that masonry hosts soluble salts and constitutes the natural habitat of fungi: hyphae live inside the masonry and are the vital part of these micro-organisms. Masonry that receives less solar energy will have lower temperatures, higher moisture contents (with the same water supply), and a greater risk of infestation. In general, north-facing walls are frequently browned by moulds.

It should be noted that the temporary protective cover (see World Heritage Centre/ ICOMOS/ ICCROM document, 23-27 September 2024, and Gelati Rehabilitation Interim Committee) is useful against rain, but is also shading a large part of the external facades which miss much solar input and cannot accumulate heat during the warm season.

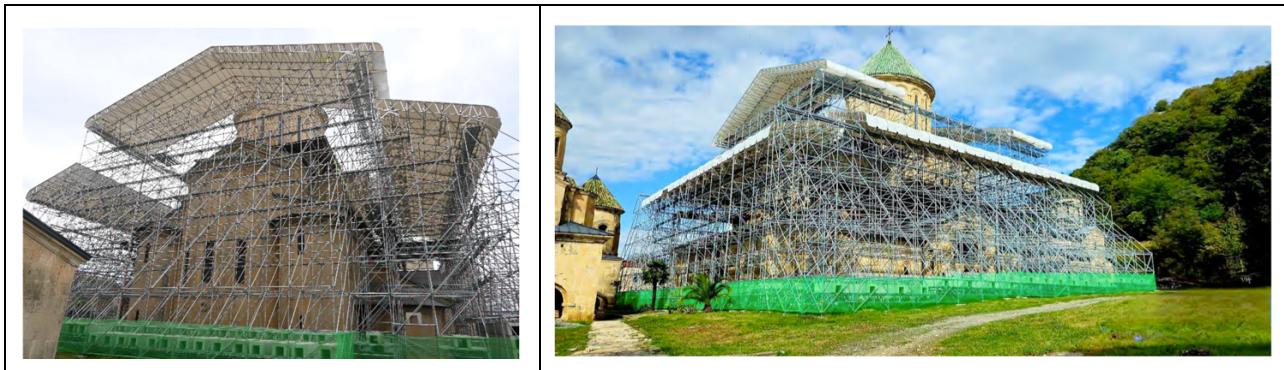


Fig.10. Temporary protective cover which constitutes a shield against rain, but also a shade stopping the solar radiation (pictures taken from the Reports World Heritage Centre/ ICOMOS/ ICCROM, and Gelati Rehabilitation Interim Committee).

It is likely that this temporary protective cover will lead to a general lowering of the masonry temperatures. The likely consequence is an increase in the masonry moisture content, which may favour the development of mould infestations. Attention should be paid to this deficit of solar energy supply.

A similar drawback was observed on the medieval monasteries of Bukovina, Romania (e.g. Voronet, Gura, Humor Homolurui and Suceava, Fig.11.). There, the enlargement of the roof pitch to protect the external frescoes from rain acted as an extended umbrella that reduced the solar income too. The lower wall temperature meant an increase of relative humidity with a parallel increase in the equilibrium moisture content in walls, resulting in internal mould infestation and blackening of all surfaces.

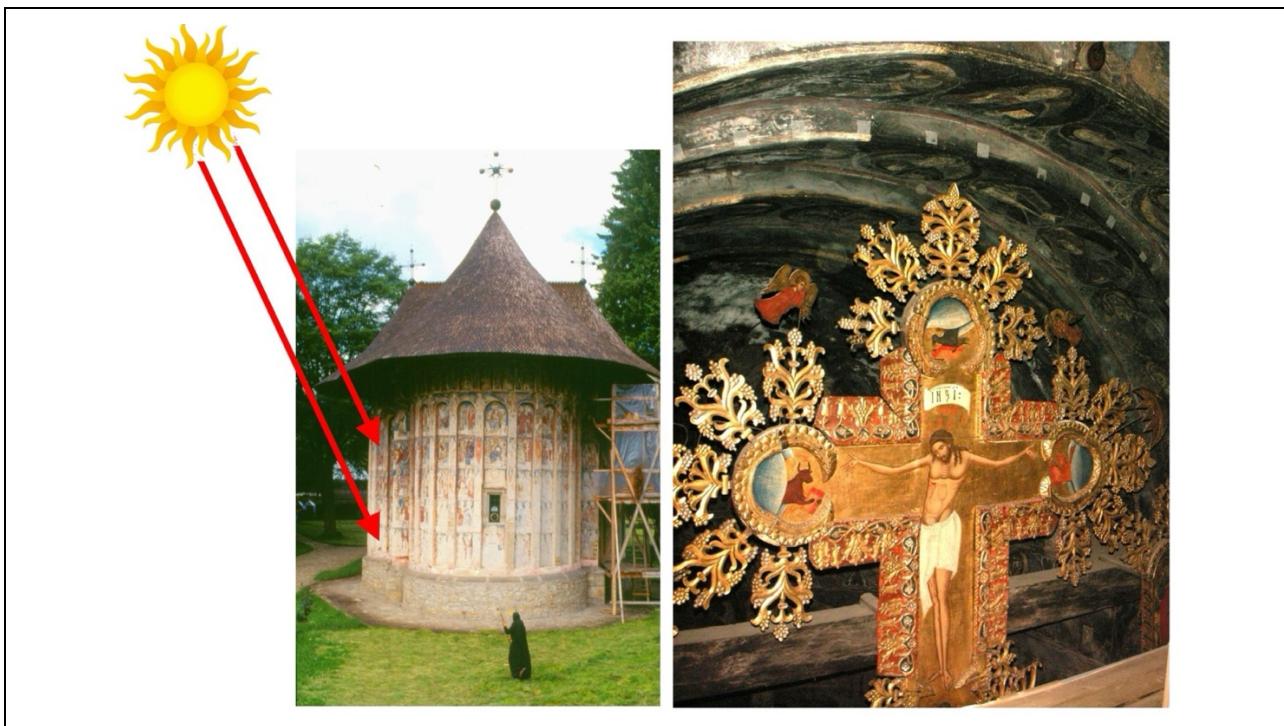


Fig.11. External and internal view of the Humor Homolurui Monastery, built 1530-35, Bucovina, Romania. The enlargement of the roof pitch and some trees planted around to shield rain lowered the solar income and the wall temperature. The interior was colonized by black fungi. The picture was taken in 2006 during the restoration, with the Cross just cleaned, and the vault behind it black.

The conclusion is that any intervention may have pros and cons, and it is essential to verify and keep the situation under control, to promptly take measures to ensure safe conservation when necessary.

Padua, 29 December 2025

